# Chapter 0

## 0.1 Introduction

This chapter includes all important aspects including the mathematical formulae useful to be successful in this physic course.

Physics becomes interesting when you relate your learning to daily activities. Therefore, the students are encouraged to reflect after each chapter, how, what, where, when the principles learnt in each chapter can be used in daily activities.

As explained in my published educational models, it is important that students do their best to develop the following six key skills while following this course. When you improve these skills while studying physics, you will reap the benefits after joining the workforce upon completion of your degree.

## 0.2 The Scientific Method

The scientific method is the single most important aspect a teacher should teach his/her students. Not only can the scientific method be applied to scientific experiments, but also to our daily activities, decision-making, and solving general problems. In our teaching, we should also attempt to improve the following activities within students through the subjects we teach. These key skills designated by the Department for Education in the United Kingdom are important aspects of the scientific method.

- To improve communication skills
- To improve number skills
- To improve IT skills
- To learn to work with others
- To improve own learning and performance
- To solve problems

The general steps involved in scientific method are as follows:

1. Name the problem or question.

2. Form an educated guess (hypothesis) of the cause of the problem and make predictions based upon the hypothesis.

3. Test your hypothesis by doing an experiment or study with proper controls.

4. Check and interpret your results

5. Report your results.

## 0.3 Solving Problems in Physics

To solve problems, you may want to implement the GUESS, or when appropriate, the GUPPESS method as follows:

| | |
|---|---|
| **G**ivens | **G**ivens |
| **U**nknowns | **U**nknowns |
| **E**quation | **P**rinciple |
| **S**ubstitution | **P**icture |
| **S**olution | **S**ubstitution |
| | **S**olution |

It is also a good idea to incorporate these steps into any problem that your teacher needs you to evaluate. It makes working through your thought processes easier.

**e.g:**

**A car accelerates at a rate of 0.60 m/s$^2$. How long does it take the car to go from 55 mi/hr to 60 mi/hr?**

**GIVENS:** $a = 0.60$ m/s$^2$; $v_0 = 55$ mi/hr; $v$ or $v_f = 60$ mi/hr

**UNKNOWNS**: t=? seconds

**PRINCIPLE**: You could use either of at least two equations to find the time when given acceleration and beginning and ending velocities: $a = \frac{\Delta v}{t}$ or $v = v_0 + at$

Part of the problem is that you need to get from mi/hr to m/s – and for that you use dimensional analysis

**PICTURE**: Probably not that helpful in this instance, but you can draw one.

**EQUATION, SUBSTITUTION, and SOLUTION**: $v = v_0 + at$

$$60 \text{ mi/hr} = 55 \text{ mi/hr} + (0.60 \text{ m/s}^2)t$$

$$60 \text{ mi/hr} - 55 \text{ mi/hr} = (0.60 \text{ m/s}^2)t$$

$$5 \text{ mi/hr} = (0.60 \text{ m/s}^2)t$$

Here is where you may want to use dimensional analysis in a side bar to convert 5 mi/hr into m/s$^2$.

$$\frac{5\text{mi}}{1\text{hr}} \times \frac{1609\text{m}}{1\text{mi}} \times \frac{1\text{hr}}{3600\text{s}} = \frac{2.2\text{m}}{1\text{s}} = 2.2\text{m}/\text{s}$$

Now substitute 2.2 m/s into the previous equation where you had 5 mi/hr.

$$5 \text{ mi/hr} = (0.60 \text{ m/s}^2)t$$

$$2.2 \text{ m/s} = (0.60 \text{ m/s}^2)t$$

$$\frac{2.2\text{m}/\text{s}}{(0.60\text{m}/\text{s}^2)} = t$$

NOTE: Follow the units! -> $\frac{2.2\text{m}}{1\text{s}} \times \frac{1\text{s}^2}{0.60\text{m}}$

**3.7 s = t**

## 0.4 Some Mathematical formulae

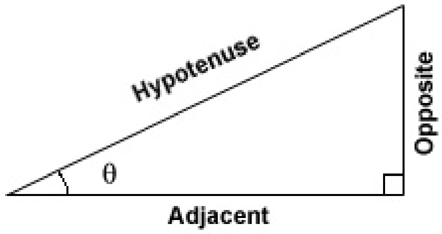
### Trigonometry

Trigonometry will become important when you study vectors.

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

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**Error Percentage Calculations**

$$\% \, Change = \frac{change}{original} \, 100 \, \%$$

$$\% \, Change = \frac{new - original}{original} \, 100 \, \%$$

**Quadratic Formula**

$$ax^2 + bx + c = 0$$

*x* is the variable and *a*, *b*, and *c* are constants

The x value is given by the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Calculus Formulae**

**Trigonometric Formulae**

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$
4. $\sin(-\theta) = -\sin\theta$
5. $\cos(-\theta) = \cos\theta$
6. $\tan(-\theta) = -\tan\theta$

7. $\sin(A+B)=\sin A\cos B+\sin B\cos A$
8. $\sin(A-B)=\sin A\cos B-\sin B\cos A$
9. $\cos(A+B)=\cos A\cos B-\sin A\sin B$
10. $\cos(A-B)=\cos A\cos B+\sin A\sin B$
11. $\sin 2\theta = 2\sin\theta\cos\theta$
12. $\cos 2\theta = \cos^2\theta-\sin^2\theta = 2\cos^2\theta-1=1-2\sin^2\theta$
13. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}=\dfrac{1}{\cot\theta}$
14. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}=\dfrac{1}{\tan\theta}$
15. $\sec\theta = \dfrac{1}{\cos\theta}$
16. $\csc\theta = \dfrac{1}{\sin\theta}$
17. $\cos(\dfrac{\pi}{2}-\theta)=\sin\theta$
18. $\sin(\dfrac{\pi}{2}-\theta)=\cos\theta$

## Differentiation Formulas

1. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
2. $\dfrac{d}{dx}(fg)=fg'+gf'$
3. $\dfrac{d}{dx}(\dfrac{f}{g})=\dfrac{gf'-fg'}{g^2}$
4. $\dfrac{d}{dx}f(g(x))=f'(g(x))g'(x)$
5. $\dfrac{d}{dx}(\sin x)=\cos x$
6. $\dfrac{d}{dx}(\cos x)=-\sin x$
7. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
8. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$
9. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$

10. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

11. $\frac{d}{dx}(e^x) = e^x$

12. $\frac{d}{dx}(a^x) = a^x \ln a$

13. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

14. $\frac{d}{dx}(Arc\sin x) = \frac{1}{\sqrt{1-x^2}}$

15. $\frac{d}{dx}(Arc\tan x) = \frac{1}{1+x^2}$

16. $\frac{d}{dx}(Arc\sec x) = \frac{1}{|x|\sqrt{x^2-1}}$

17. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ *Chain Rule*

**Integration Formulae**

1. $\int a\, dx = ax + C$

2. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

3. $\int \frac{1}{x} dx = \ln|x| + C$

4. $\int e^x\, dx = e^x + C$

5. $\int a^x dx = \frac{a^x}{\ln a} + C$

6. $\int \ln x\, dx = x \ln x - x + C$

7. $\int \sin x\, dx = -\cos x + C$

8. $\int \cos x\, dx = \sin x + C$

9. $\int \tan x\, dx = \ln|\sec x| + C \text{ or } -\ln|\cos x| + C$

10. $\int \cot x\, dx = \ln|\sin x| + C$

11. $\int \sec x\, dx = \ln|\sec x + \tan x| + C$

12. $\int \csc x\, dx = \ln|\csc x - \cot x| + C$

13. $\int \sec^2 x\, dx = \tan x + C$

14. $\int \sec x \tan x\, dx = \sec x + C$

15. $\int \csc^2 x\,dx = -\cot x + C$

16. $\int \csc x \cot x\,dx = -\csc x + C$

17. $\int \tan^2 x\,dx = \tan x - x + C$

18. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} Arc\tan\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{\sqrt{a^2 - x^2}} = Arc\sin\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} Arc\sec\frac{|x|}{a} + C = \frac{1}{a} Arc\cos\left|\frac{a}{x}\right| + C$

**Formulae and Theorems**

Definition of Limit: Let $f$ be a function defined on an open interval containing $c$ (except possibly at $c$) and let $L$ be a real number. Then $\lim_{x \to a} f(x) = L$ means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

1b. A function $y = f(x)$ is continuous at $x = a$ if

i). *f(a)* exists

ii). $\lim_{x \to a} f(x)$ exists

iii). $\lim_{x \to a} = f(a)$

Even and Odd Functions

1. A function $y = f(x)$ is even if $f(-x) = f(x)$ for every $x$ in the function's domain.
   Every even function is symmetric about the y-axis.

2. A function $y = f(x)$ is odd if $f(-x) = -f(x)$ for every $x$ in the function's domain.
   Every odd function is symmetric about the origin.

Periodicity

A function $f(x)$ is periodic with period $p\ (p > 0)$ if $f(x + p) = f(x)$ for every value of $x$.

*Note*: The period of the function $y = A\sin(Bx + C)$ or $y = A\cos(Bx + C)$ is $\frac{2\pi}{|B|}$.

The amplitude is $|A|$. The period of $y = \tan x$ is $\pi$.

<u>Intermediate-Value Theorem</u>

A function $y = f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$.

*Note*: If $f$ is continuous on $[a,b]$ and $f(a)$ and $f(b)$ differ in sign, then the equation $f(x) = 0$ has at least one solution in the open interval $(a,b)$.

<u>Limits of Rational Functions as $x \to \pm\infty$</u>

i). $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = 0$ if the degree of $f(x) <$ the degree of $g(x)$

*Example*: $\lim_{x \to \infty} \frac{x^2 - 2x}{x^3 + 3} = 0$

ii). $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}$ is infinite if the degrees of $f(x) >$ the degree of $g(x)$

*Example*: $\lim_{x \to \infty} \frac{x^3 + 2x}{x^2 - 8} = \infty$

iii). $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}$ is finite if the degree of $f(x) =$ the degree of $g(x)$

*Example*: $\lim_{x \to \infty} \frac{2x^2 - 3x + 2}{10x - 5x^2} = -\frac{2}{5}$

<u>Horizontal and Vertical Asymptotes</u>

1. A line $y = b$ is a <u>horizontal asymptote</u> of the graph $y = f(x)$ if either $\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$.

2. A line $x = a$ is a <u>vertical asymptote</u> of the graph $y = f(x)$ if either $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} = \pm\infty$.

<u>Average and Instantaneous Rate of Change</u>

i). <u>*Average Rate of Change*</u>: If $(x_0, y_0)$ and $(x_1, y_1)$ are points on the graph of $y = f(x)$, then the average rate of change of $y$ with respect to $x$ over the interval $[x_0, x_1]$ is $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{y_1 - y_0}{x_1 - x_0} = \dfrac{\Delta y}{\Delta x}$.

ii). <u>*Instantaneous Rate of Change*</u>: If $(x_0, y_0)$ is a point on the graph of $y = f(x)$, then the instantaneous rate of change of $y$ with respect to $x$ at $x_0$ is $f'(x_0)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

<u>The Number ***e*** as a limit</u>

i). $\lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n = e$

ii). $\lim_{n \to 0} \left(1 + \frac{n}{1}\right)^{\frac{1}{n}} = e$

<u>Rolle's Theorem</u>

If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$ such that $f(a) = f(b)$, then there is at least one number $c$ in the open interval $(a, b)$ such that $f'(c) = 0$.

<u>Mean Value Theorem</u>

If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then there is at least one number $c$ in $(a, b)$ such that $\dfrac{f(b) - f(a)}{b - a} = f'(c)$.

Extreme-Value Theorem

If $f$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both a maximum and minimum on $[a,b]$.

To find the maximum and minimum values of a function $y = f(x)$, locate

1. the points where $f'(x)$ is zero or where $f'(x)$ fails to exist.
2. the end points, if any, on the domain of $f(x)$.

*Note*: These are the only candidates for the value of $x$ where $f(x)$ may have a maximum or a minimum.

Let $f$ be differentiable for $a < x < b$ and continuous for a $a \le x \le b$,

1. If $f'(x) > 0$ for every $x$ in $(a,b)$, then $f$ is increasing on $[a,b]$.
2. If $f'(x) < 0$ for every $x$ in $(a,b)$, then $f$ is decreasing on $[a,b]$.

**0.5 Errors**

Measurement is the basis of scientific study. All measurements are, however, approximate values (not true values) within the limitation of a measuring device, measuring environment, process of measurement and human error. We seek to minimize uncertainty and hence error to the extent possible.

Further, there is an important aspect of reporting measurement. It should be consistent, systematic and revealing in the context of accuracy and precision. We must understand that an error in basic quantities propagate through mathematical formula leading to compounding of errors and misrepresentation of quantities.

Errors are broadly classified into two categories:

- Systematic error
- Random error

A systematic error impacts “accuracy” of the measurement. Accuracy means how close is the measurement with respect to “true” value. A “true” value of a quantity is a measurement, when errors on all accounts are minimized. We should distinguish “accuracy” of measurement with “precision” of measurement, which is related to the ability of an instrument to measure values with greater details (divisions).

The measurement of a weight on a scale with marking in kg is 79 kg, whereas measurement of the same weight on a different scale having further divisions in hectogram is 79.3 kg. The later weighing scale is more precise. The precision of measurement of an instrument, therefore, is a function of the ability of an instrument to read smaller divisions of a quantity.

Summary:

1. True value of a quantity is an "unknown". We cannot know the true value of a quantity, even if we have measured it by chance, as we do not know the exact value of error in measurement. We can only approximate true value with greater accuracy and precision.
2. An accepted "true" measurement of a quantity is a measurement, when errors on all accounts are minimized.
3. "Accuracy" means how close the measurement is with respect to "true" measurement. It is associated with systematic error.
4. "Precision" of measurement is related to the ability of an instrument to measure values in greater detail. It is associated with random error.

Systematic error

A systematic error results due to faulty measurement practices. An error of this type is characterized by deviation in one direction from the true value. Itt means that the error is introduced, which is either less than or greater than the true value. Systematic error impacts the accuracy of measurement – not the precision of the measurement.

Systematic error results from:

1. faulty instrument
2. faulty measuring process and
3. personal bias

Clearly, this type of error cannot be minimized or reduced by repeated measurements. A faulty machine, for example, will not improve accuracy of measurement by repeating measurements.

Instrument error

A zero error, for example, is an instrument error, which is introduced in the measurement consistently in one direction. A zero error results when the zero mark of the scale does not match with pointer. We can realize this with the weighing instrument (scale) we use in our home. Often, the pointer is off the zero mark of the scale. Moreover, the scale may in itself be not uniformly marked or may not be properly calibrated. In vernier calipers, the nine divisions of main scale should be exactly equal to ten divisions of vernier scale. In a nutshell, we can say that the instrument error occurs due to faulty design of the instrument. We can minimize this error by replacing the instrument or by making a change in the design of the instrument.

Procedural error

A faulty measuring process may include an inappropriate physical environment, procedural mistakes and a lack of understanding of the process of measurement. For example, if we are studying the magnetic effect of current, then it would be erroneous to conduct the experiment in a place where strong currents are flowing nearby. Similarly, while taking the temperature of a human body, it is important to know which of the human parts is more representative of body temperature.

This type of error can be minimized by periodically assessing the measurement process and improvising the system.  Consulting a subject expert or simply conducting an audit of the measuring process is also helpful in light of new facts and advancements.

Personal bias

A personal bias is introduced by human habits, which are not conducive for accurate measurement. Consider, for example, the reading habit of a person. He or she may have the habit of reading scales from an inappropriate distance and from an oblique direction. The measurement, therefore, includes error because of parallax.

**Parallax**

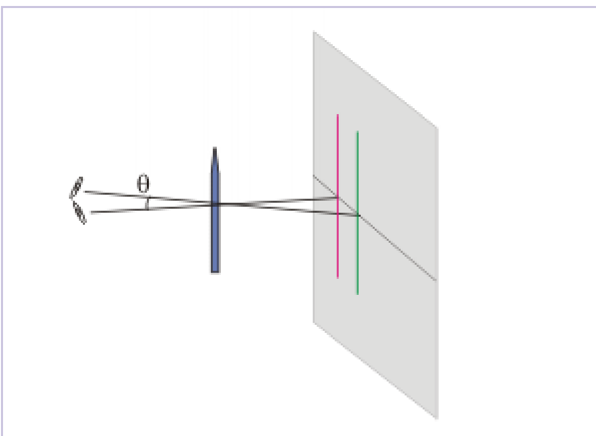


**Figure 0.1:** The position of pencil changes with respect to a mark on the background.

We can appreciate the importance of parallax by just holding a finger (pencil) in the hand, which is stretched horizontally. We keep the finger in front of our eyes against some reference marking in the background. Now, we look at the finger by closing one eye at a time and note the relative displacement of the finger with respect to the mark in the static background. We can do this experiment any time as shown in the figure above. The parallax results due to the angle at which we look at the object.

It is important that we read position of a pointer or a needle on a scale normally to avoid error because of parallax.

**Parallax**

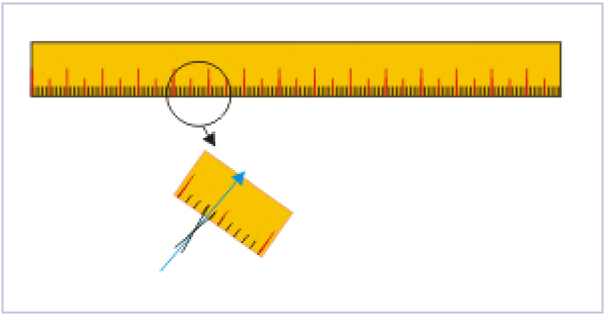

**Figure 0.2:** Parallax error is introduced as we may read values at an angle.

Random errors

Random error unlike systematic error is not unidirectional. Some of the measured values are greater than true value; some are less than true value. The errors introduced are sometimes positive and sometimes negative with respect to true value. It is possible to minimize this type of error by repeating measurements and applying statistical techniques to get closer value to the true value.

Another distinguishing aspect of random error is that it is not biased. It is there because of the limitation of the instrument in hand and the limitation on the part of human ability. No human being can repeat an action in exactly the same manner. Hence, it is likely that the same person reports different values with the same instrument, which measures the quantity correctly.

Least count error

Least count error results due to the inadequacy of resolution of the instrument. We can understand this in the context of least count of a measuring device. The least count of a device is equal to the smallest division on the scale. Consider the meter scale that we use. What is its least count? Its smallest division is in millimeter (mm). Hence, its least count is 1 mm i.e. $10^{-3}$ m i.e. 0.001 m. Clearly, this meter scale can be used to measure length from $10^{-3}$ m to 1 m. It is worth to know that least count of a vernier scale is $10^{-4}$ m and that of screw gauge and spherometer is $10^{-5}$ m.

Returning to the meter scale, we have the dilemma of limiting ourselves to the exact measurement up to the precision of marking or should be limited to a step before. For example, let us read the measurement of a piece of a given rod. One end of the rod exactly matches with the zero of scale. The other end lies at the smallest markings at 0.477 m (= 47.7 cm = 477 mm). We may argue that measurement should be limited to the marking, which can be definitely relied. If so, then we would report the length as 0.47 m, because we may not be definite about millimeter reading.

This is, however, unacceptable, as we are sure that length consists of some additional length – only thing that we may err as the reading might be 0.476 m or 0.478 m instead of 0.477 m. There is a definite chance of error due to limitation in reading such small divisions. We would be more precise and accurate by reporting measurement as 0.477 ± some agreed level of anticipated error. Generally, the accepted level of error in reading the smallest division is considered to be half of the least count. Hence, the reading would be:

$\Rightarrow x = 0.477 \pm 0.0005 m$

If we report the measurement in centimeter,

$\Rightarrow x = 47.7 \pm 0.05 cm$

If we report the measurement in millimeter,

$\Rightarrow x = 477 \pm 0.5 mm$

Mean value of measurements

It has been pointed out that random error, including that of least count error, can be minimized by repeating measurements. It is so because errors are not unidirectional. If we take average of the measurements from the repeated measurements, it is likely that we minimize error by canceling out errors in opposite directions.

Here, we are implicitly assuming that measurement is free of “systematic errors”. The averaging of the repeated measurements, therefore, gives the best estimate of “true” value. As such, average or mean value ( $a_m$ ) of the measurements (excluding "of- beat" measurements) is the notional “true” value of the quantity being measured. In fact, it is reported as true value, being our best estimate.

**Error Propagation**

In this module, we shall introduce some statistical analysis techniques to improve our understanding about error and enable reporting of error in the measurement of a quantity. There are three related approaches, which involves measurement of:

Absolute error

Relative error

Percentage error

Absolute error

The absolute error is the magnitude of error as determined from the difference of measured value from the mean value of the quantity. The important thing to note here is that absolute error is concerned with the magnitude of error – not the direction of error. For a particular nth measurement,

$$|\Delta x_n| = |x_n - x_n|$$

where "$x_m$" is the mean or average value of measurements and "$x_n$" is the nth instant of measurement.

In order to calculate a few absolute values, we consider a set of measured data for the length of a given rod. Note that we are reporting measurements in centimeter.

$x_1 = 47.7$ cm, $x_2 = 47.5$ cm, $x_3$ =47.8 cm, $x_4 = 47.4$ cm and $x_5 = 47.7$ cm

The mean value of length is,

$\Rightarrow x_m = 47.62$ cm

It is evident from the individual values that the least count of the scale (smallest division) is 0.001 m = 0.1 cm. For this reason, we limit mean value to the first decimal place. Hence, we round off the last but one digit as:

$x_m$=47.6cm

This is the mean or true value of the length of the rod. Now, absolute error of each of the five measurements are,

$|\Delta x_1|=|x_m - x_1| = |47.6-47.7| = |-0.1 = .1$ cm

$|\Delta x_2|=|x_m - x_2| = |47.6-47.5| = |0.1| =0.1$ cm

$|\Delta x_3|=|x_m - x_3| = |47.6-47.8| = |-0.2| = 0.2$ cm

$|\Delta x_4|=|x_m - x_4| = |47.6-47.4| = |0.2| = 0.2$ cm

$|\Delta x_5|=|x_m - x_5| = |47.6-47.7| = |-0.1| = 0.1$ cm

**Mean absolute error**

Earlier, it was stated that a quantity is measured with a range of error specified by half the least count. This is a generally accepted range of error. Here, we shall work to calculate the range of the error, based on the actual measurements and not go by any predefined range of error as that of generally accepted range of error. This means that we want to determine the range of error, which is based on the deviations in the reading from the mean value.

Absolute error associated with each measurement tells us how far the measurement can be off the mean value. The absolute errors calculated, however, may be different. Now the question is, which of the absolute errors should be taken for our consideration? We take the average of the absolute error:

The value of measurement, now, will be reported with the range of error as:

$x = x_m \pm \Delta x_m$

Extending this concept of defining range to the earlier example, we have,

$\Rightarrow \Delta x_m = 0.1\text{cm}.$

We should note here that we have rounded the result to reflect that the error value that has same precision as that of measured value. The value of the measurement with the range of error, then, is :

$\Rightarrow x = 47.6 \pm 0.1\text{cm}$

A plain reading of above expression is “the length of the rod lies in between 47.5 cm and 47.7 cm”. For all practical purpose, we shall use the value of x = 47.6 cm with caution, in that this quantity involves an error of the magnitude of “0.1 cm” in either direction.